24th Feb. 2021 | Shift - 2
MATHEMATICS

JEE | NEET | Foundation

Topic :- 3D

Subtopic:- Length & food of 1st image of pt w.r.t. plane (M176)

Level :- Medium

1. Let $a, b \in R.$ If the mirror image of the point $P(a,6,9)$ with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

(1) 86 (2) 88 (3) 84 (4) 90

माना $a, b \in R.$ यदि बिन्दु $P(a,6,9)$ का रेखा, $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ में दर्पण प्रतिबिम्ब $(20, b, -a-9)$, है तो $|a+b|$ बराबर है:

(1) 86 (2) 88 (3) 84 (4) 90

Ans. **(2)**

Sol. P(a, 6, 9), Q (20, b, −a−9)

mid point of $PQ = \left(\frac{a+20}{2}, \frac{b+6}{2}, -\frac{a}{2}\right)$

lie on line

$$\frac{\frac{a+20}{2}-3}{7}=\frac{\frac{b+6}{2}-2}{5}=\frac{-\frac{a}{2}-1}{-9}$$

$$\frac{a+20-6}{14}=\frac{b+6-4}{10}=\frac{-a-2}{-18}$$

$$\frac{a+14}{14}=\frac{a+2}{18}$$

$$18a+252=14a+28$$

$$4a=-224$$

$$\boxed{a=-56}$$

$$\frac{b+2}{10}=\frac{a+2}{18}$$

$$\frac{b+2}{10}=\frac{-54}{18}$$

$$\frac{b+2}{10}=-3 \Rightarrow b=-32$$

$$|a+b|=|-56-32|=88$$

Topic :- I.I.

Subtopic:- integration between substitution (M221)

Level :- Medium

2. Let f be a twice differentiable function defined on R such that $f(0)=1, f'(0)=2$ and $f'(x)\neq 0$ for all x∈R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0,$ for all x∈R then the value of $f(1)$ lies in the interval:

(1) (9, 12) (2) (6, 9) (3) (3, 6) (4) (0, 3)

माना R पर एक फलन f दो बार अवकलनीय है, जिसके लिए $f(0)=1, f'(0)=2$ तथा $f'(x)\neq 0$ सभी x∈R के लिए है।
यदि $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0,$ ∀x∈R है तो $f(1)$ का मान जिस अन्तराल में है, वह है:

(1) (9, 12) (2) (6, 9) (3) (3, 6) (4) (0, 3)

Ans. (2)

Sol. Given $f(x) f''(X) - (f'(x))^2 = 0$

Let $h(x) = \dfrac{f(x)}{f'(x)}$

$\Rightarrow h'(x) = 0$ $\Rightarrow h(x) = k$

$\Rightarrow \dfrac{f(x)}{f'(x)} = k$ $\Rightarrow f(x) = k f'(x)$

$\Rightarrow f(0) = k f'(0)$ $\Rightarrow 1 = k(2) \Rightarrow k = \dfrac{1}{2}$

Now $f(x) = \dfrac{1}{2} f'(x) \Rightarrow \int 2dx = \int \dfrac{f'(x)}{f(x)}dx$

$\Rightarrow 2x = ln|f(x)| + C$
As $f(0) = 1 \Rightarrow C = 0$
$\Rightarrow 2x = ln|f(X)| \Rightarrow f(x) = \pm e^{2x}$
As $f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$

Topic :- I.T.F

Subtopic:- Domain Range (M127)

Level :- Medium

3. A possible value of $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$ is:

(1) $\dfrac{1}{2\sqrt{2}}$ (2) $\dfrac{1}{\sqrt{7}}$ (3) $\sqrt{7}-1$ (4) $2\sqrt{2}-1$

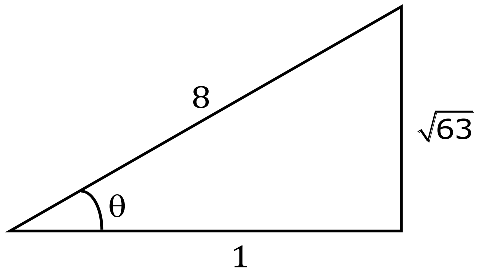
$\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ का एक सम्भावित मान है:

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{7}}$ (3) $\sqrt{7}-1$ (4) $2\sqrt{2}-1$

Ans. **(2)**

Sol. $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)=\theta$ $\sin\theta=\frac{\sqrt{63}}{8}$



$$\cos\theta=\frac{1}{8}$$

$$2\cos^2\frac{\theta}{2}-1=\frac{1}{8}$$

$$\cos^2\frac{\theta}{2}=\frac{9}{16}$$

$$\cos\frac{\theta}{2}=\frac{3}{4}$$

$$\frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}}=\frac{3}{4}$$

$$\tan\frac{\theta}{4}=\frac{1}{\sqrt{7}}$$

Topic :- Probability

Subtopic:- sample space detinition (M225)

Level :- Medium

4. The probability that two randomly selected subsets of the set {1,2,3,4,5} have exactly two elements in their intersection, is:

(1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$ (3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

समुच्चय {1,2,3,4,5} से दो यादृच्छिक चुने गए उपसमुच्चयों के सर्वनिष्ठ में ठीक दो अवयव होने की प्रायिकता है:

(1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$ (3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

Ans. (2)

Sol. Required probability

$$= \frac{{}^5C_2 \times 3^3}{4^5}$$

$$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$$

Topic :- 3D

Subtopic:- Mixed (M178)

Level :- Easy

5. The vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$, and the point (1,0,2) is :

(1) $\vec{r}.(\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$ (2) $\vec{r}.(\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(3) $\vec{r}.(3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$ (4) $\vec{r}.(\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

समतलों $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ तथा $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$ के प्रतिच्छेदन से तथा बिन्दु (1,0,2) से होकर जाने वाले समतल का सदिश समीकरण है:

(1) $\vec{r}.(\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$ (2) $\vec{r}.(\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(3) $\vec{r}.(3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$ (4) $\vec{r}.(\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

Ans. (2)

Sol. Plane passing through intersection of plane is

$\{\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1\}+\lambda\{\vec{r}\cdot(\hat{i}-2\hat{j})+2\}=0$

Passes through $\hat{i}+2\hat{k}$, we get

$(3-1)+\lambda(1+2)=0 \Rightarrow \lambda=-\frac{2}{3}$

Hence, equation of plane is $3\{\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1\}-2\{\vec{r}\cdot(\hat{i}-2\hat{j})+2\}=0$

$\Rightarrow \quad \vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=7$

Topic :- Tangent & normal

Subtopic:- T & n when slaeris known (M283)

Level :- Easy

6. If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1$, then the co-ordinates of P are :

(1) (−2, 8) (2) (1, 5) (3) (3, 13) (4) (2, 8)

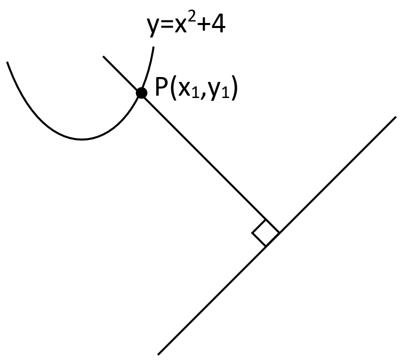
यदि P परवलय $y=x^2+4$ पर एक ऐसा बिन्दु है जो सरलरेखा $y=4x-1$, के निकटतम है तो P के निर्देशांक है:

(1) (−2, 8) (2) (1, 5) (3) (3, 13) (4) (2, 8)

Ans. **(4)**

Sol. $\frac{dy}{dx}\Big|_p = 4$

$\therefore 2x_1 = 4$



$\Rightarrow x_1 = 2$

$\therefore$ Point will be (2, 8)

Topic :- S & P (Progerssion)

Subtopic:- Mixed (M22)

Level :- Medium

7. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a,c),(2,b)$ and (a,b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α,β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is:

(1) $\frac{71}{256}$ (2) $-\frac{69}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{71}{256}$

माना a, b, c एक समान्तर श्रेणी में है। माना त्रिभुज जिसके शीर्ष बिन्दु $(a,c),(2,b)$ तथा (a,b) है, का केन्द्रक $\left(\frac{10}{3},\frac{7}{3}\right)$. है। यदि समीकरण $ax^2+bx+1=0$, के मूल α तथा β है, तो $\alpha^2+\beta^2-\alpha\beta$ का मान है:

(1) $\frac{71}{256}$ (2) $-\frac{69}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{71}{256}$

Ans. **(4)**

Sol. 2b = a + c

$\frac{2a+2}{3}=\frac{10}{3}$ and $\frac{2b+c}{3}=\frac{7}{3}$

a = 4, $\left.\begin{matrix}2b+c=7\\2b-c=4\end{matrix}\right\}$, solving

$b=\frac{11}{4}$

$c=\frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2+\frac{11}{4}x+1=0$

$\therefore$ The value of $(\alpha+\beta)^2-3\alpha\beta=\frac{121}{256}-\frac{3}{4}=-\frac{71}{256}$

Topic :- D. I.

Subtopic:- Stenderd integral (M120)

Level :- Medium

8. The value of the integral, $\int_1^3\left[x^2-2x-2\right]dx$, where [x] denotes the greatest integer less than or equal to x, is:

(1) –4 (2) –5 (3) $-\sqrt{2}-\sqrt{3}-1$ (4) $-\sqrt{2}-\sqrt{3}+1$

समाकलन $\int_1^3 \left[x^2 - 2x - 2\right]dx$ का मान, जबकि [x], महत्तम पूर्णांक $\le x$ है, हैं :

(1) -4 (2) -5 (3) $-\sqrt{2}-\sqrt{3}-1$ (4) $-\sqrt{2}-\sqrt{3}+1$

Ans. **(3)**

Sol. $I = \int_1^3 -3dx + \int_1^3 \left[(x-1)^2\right]dx$

Put x −1 = t ; dx = dt

$I = (-6) + \int_0^2 \left[t^2\right]dt$

$I = -6 + \int_0^1 0\,dt + \int_1^{\sqrt{2}} 1\,dt + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dt + \int_{\sqrt{3}}^{2} 3\,dt$

$I = -6 + \left(\sqrt{2}-1\right) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$I = -1 - \sqrt{2} - \sqrt{3}$

Topic :- Monotonocity

Subtopic:- Finding intervals of monotonocity (M293)

Level :- Medium

9. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4 \end{cases}$$

Let $A = \{x \in R : f$ is increasing}. Then A is equal to :

(1) $(-5,-4)\cup(4,\infty)$ (2) $(-5,\infty)$

(3) $(-\infty,-5)\cup(4,\infty)$ (4) $(-\infty,-5)\cup(-4,\infty)$

माना $f : \mathbf{R} \to \mathbf{R}$

$$f(x) = \begin{cases} -55x, & \text{यदि } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{यदि } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{यदि } x > 4 \end{cases}$$

द्वारा परिभाषित है। माना $A = \{x \in R : f$ वर्धमान है} तो A बराबर है:

(1) $(-5,-4)\cup(4,\infty)$ (2) $(-5,\infty)$

(3) $(-\infty,-5)\cup(4,\infty)$ (4) $(-\infty,-5)\cup(-4,\infty)$

Ans. (1)

Sol. $f(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x^2 - x - 20) & ; \quad -5 < x < 4 \\ 6(x^2 - x - 6) & ; \quad x > 4 \end{cases}$

$f(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x-5)(x+4) & ; \quad -5 < x < 4 \\ 6(x-3)(x+2) & ; \quad x > 4 \end{cases}$

Hence, f(x) is monotonically increasing in interval $(-5, -4)\cup(4, \infty)$

Topic :- T & N

Subtopic:- T & n when slope is known (M283)

Level :- Easy

10. If the curve $y = ax^2 + bx + c, x \in R,$ passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a,b,c are :

(1) a =1, b=1, c=0 (2) a= −1, b=1, c =1

(3) a =1, b=0, c =1 (4) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

यदि वक्र $y = ax^2 + bx + c, x \in R$ बिन्दु (1,2) से होकर जाता है तथा मूलबिन्दु पर इसकी स्पर्श रेखा $y = x$ है, तो, a,b,c के संभावित मान है:

(1) a =1, b=1, c=0 (2) a= −1, b=1, c =1

(3) a =1, b=0, c =1 (4) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

Ans. (1)

Sol. 2 = a + b + c ......(i)

$\frac{dy}{dx} = 2ax + b \Rightarrow \left.\frac{dy}{dx}\right|_{(0,0)} = 1$

$\Rightarrow b = 1 \Rightarrow a + c = 1$

(0,0) lie on curve

$\therefore$ c=0, a=1

Topic :- Set & Relation

Subtopic:- Mathematical Resoning

Level :- Easy

11. The negation of the statement

$\sim p \wedge (p \vee q)$ is :

(1) ~p ∧ q (2) p∧~q (3) ~p ∨ q (4) p∨~q

कथन $\sim p \wedge (p \vee q)$ का निषेधन है:

(1) ~p ∧ q (2) p∧~q (3) ~p ∨ q (4) p∨~q

Ans. (4)

Sol.

p	q	~ p	p ∨ q	(~ p) ∧ (p∨ q)	~ q	p∨ ~ q
T	T	F	T	F	F	T
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	T	T

$\therefore \sim p \wedge (p \vee q) \equiv p \vee \sim q$

Topic :- Determinant

Subtopic:- Crammer's Rule (M190)

Level :- Medium

12. For the system of linear equations:

$x-2y=1, x-y+kz=-2, ky+4z=6, k \in \mathbf{R}$

consider the following statements:

(A) The system has unique solution if $k \neq 2, k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct?

(1) (B) and (E) only (2)(C) and (D) only

(3) (A) and (D) only (4) (A) and (E) only

रैखिक समीकरण निकाय

$x-2y=1, x-y+kz=-2, ky+4z=6, k\in \mathbf{R}$

के लिए, नीचे दिए कथनों पर विचार कीजिए:

(A) निकाय का केवल एक हल है, यदि $k\neq 2, k\neq -2$ है।

(B) निकाय का केवल एक हल है, यदि $k=-2$ है।

(C) निकाय का केवल एक हल है, यदि $k=2$ है।

(D) निकाय का कोई हल नहीं है, यदि $k=2$ है।

(E) निकाय के अनन्त हल हैं, यदि $k\neq -2$ है।

तो निम्न कथनों में कौन से सत्य है।

(1) केवल (B) तथा (E)
(2) केवल (C) तथा (D)
(3) केवल (A) तथा (D)
(4) केवल (A) तथा (E)

Ans. (3)

Sol. $x-2y+0.z=1$

$x-y+kz=-2$

$0.x+ky+4z=6$

$$\Delta=\begin{vmatrix}1 & -2 & 0\\ 1 & -1 & k\\ 0 & k & 4\end{vmatrix} = 4-k^2$$

For unique solution $4-k^2\neq 0$

$\boxed{k\neq \pm 2}$

For k=2

$x-2y+0.z=1$

$x-y+2z=-2$

$0.x+2y+4z=6$

$$\Delta x=\begin{vmatrix}1 & -2 & 0\\ -2 & -1 & 2\\ 6 & 2 & 4\end{vmatrix}=(-8)+2[-20]$$

$\Delta x=-48\neq 0$

For k=2 $\Delta x\neq 0$

For K=2; The system has no solution

Topic :- Conic (parabola, ellipse, hyperbola)

Subtopic:- Mixed (M280)

Level :- Easy

13. For which of the following curves, the line $x+\sqrt{3}y=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$?

(1) $x^2 + 9y^2 = 9$ (2) $2x^2 - 18y^2 = 9$

(3) $y^2 = \frac{1}{6\sqrt{3}}x$ (4) $x^2 + y^2 = 7$

बिन्दु $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$ पर, रेखा $x+\sqrt{3}y=2\sqrt{3}$, निम्न में से किस वक्र को स्पर्श करती है?

(1) $x^2 + 9y^2 = 9$ (2) $2x^2 - 18y^2 = 9$

(3) $y^2 = \frac{1}{6\sqrt{3}}x$ (4) $x^2 + y^2 = 7$

Ans. (1)

Sol. Tangent to $x^2 + 9y^2 = 9$ at point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$ is x $\left|\frac{3\sqrt{3}}{2}\right| + 9y\left(\frac{1}{2}\right) = 9$

$3\sqrt{3}x + 9y = 18 \Rightarrow x + \sqrt{3}y = 2\sqrt{3}$

$\Rightarrow$ option (1) is true.

Topic :- Set & Reation

Subtopic:- Mathematical Reasoning

Level :- Medium

14. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/ hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is:

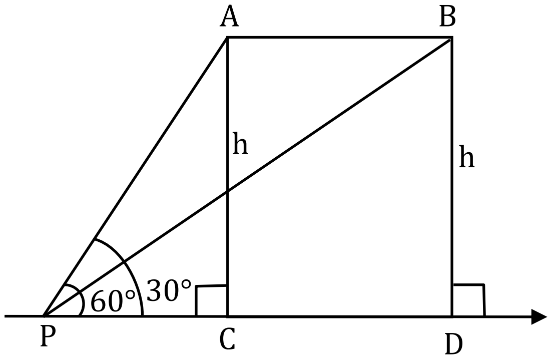
(1) $1200\sqrt{3}$m (2) $1800\sqrt{3}$m (3) $3600\sqrt{3}$m (4) $2400\sqrt{3}$m

धरती पर एक बिन्दु A से एक जैट का उन्नयन कोण 60° है। 432 km/hour की गति से 20 सैकेंड की उड़ान के बाद उन्नयन कोण 30° हो जाता है। यदि जेट एक स्थिर ऊँचाई पर उड़ रहा है, तो उसकी ऊँचाई है:

(1) $1200\sqrt{3}$m (2) $1800\sqrt{3}$m (3) $3600\sqrt{3}$m (4) $2400\sqrt{3}$m

Ans. **(1)**

Sol.



$v = 432 \times \frac{1000}{60 \times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In ΔPAC

$\tan 60° = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$

In ΔPBD

$\tan 30° = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$

PD = PC + CD

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

$h = 1200\sqrt{3}$ meter

Topic :- Set & Relation

Subtopic:- Mathematical Reasoning

Level :- Medium

15. For the statements p and q, consider the following compound statements:

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(1) (a) is a tautology but not (b)

(2) (a) and (b) both are not tautologies.

(3) (a) and (b) both are tautologies.

(4) (b) is a tautology but not (a).

कथनों p तथा q के लिए, निम्न मिश्र कथनों पर विचार कीजिए:

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

तो निम्न कथनों में से कौन–सा कथन सत्य है ?

(1) (a) एक पुनरूक्ति है, परन्तु (b) नहीं है
(2) (a) तथा (b) दोनों पुनरूक्तियाँ नहीं है
(3) (a) तथा (b) दोनों पुनरूक्तियाँ है
(4) (b) एक पुनरूक्ति है, परन्तु (a) नहीं है

Ans. **(3)**

Sol. **(a)**

p	q	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim p$	$(\sim q) \wedge (p \rightarrow q) \rightarrow \sim p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

(a) is tautologies

(b)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$((p \vee q) \wedge \sim p) \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(b) is tautologies

$\therefore$ a & b are both tautologies.

Topic :- Matrix

Subtopic:- Mixed (M185)

Level :- Medium

16. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2 B^2 - B^2 A^2) X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

(1) a unique solution
(2) exactly two solutions
(3) infinitely many solutions
(4) no solution

माना A तथा B दो 3×3 वास्तविक आव्यूह है जबकि A सममित है तथा B विषम सममित आव्यूह है। तो रैखिक समीकरण निकाय, $(A^2 B^2 - B^2 A^2) X = O$ जबकि X एक 3×1 अज्ञात चरों का स्तम्भ आव्यूह है तथा O एक 3×1 शून्य आव्यूह है:

(1) का केवल एक हल है (2) के ठीक दो हल हैं

(3) के अनन्त हल है (4) का कोई भी हल नहीं है

Ans. **(3)**

Sol. $A^T = A, B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $ay + bz = 0$ ...(1)

$-ax + cz = 0$ ...(2)

$-bx - cy = 0$ ...(3)

From equation 1,2,3

$\Delta = 0$ & $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$\therefore$ equation have infinite number of solution

Topic :- Binomial Theorem

Subtopic:- Problem based on binomial coeff. (M27)

Level :- Medium

17. If $n \geq 2$ is a positive integer, then the sum of the series

$^{n+1}C_2 + 2\left(^2C_2 + ^3C_2 + ^4C_2 + \ldots + ^nC_2\right)$ is :

(1) $\dfrac{n(n+1)^2(n+2)}{12}$ (2) $\dfrac{n(n-1)(2n+1)}{6}$

(3) $\dfrac{n(n+1)(2n+1)}{6}$ (4) $\dfrac{n(2n+1)(3n+1)}{6}$

यदि $n \geq 2$ एक धनात्मक पूर्णांक है, तो श्रेणी

$^{n+1}C_2 + 2\left(^2C_2 + ^3C_2 + ^4C_2 + \ldots + ^nC_2\right)$ का योग है:

(1) $\dfrac{n(n+1)^2(n+2)}{12}$ (2) $\dfrac{n(n-1)(2n+1)}{6}$

(3) $\frac{n(n+1)(2n+1)}{6}$ (4) $\frac{n(2n+1)(3n+1)}{6}$

Ans. (3)

Sol. ${}^{2}C_2 = {}^{3}C_3$

$S = {}^{3}C_3 + {}^{3}C_2 + ... + {}^{n}C_2 = {}^{n+1}C_3$

$\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \frac{(n+1)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$

$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)(2n+1)}{6}$

Topic :- Differential Equaiton

Subtopic:- Linear first order (137)

Level :- Medium

18. If a curve $y = f(x)$ passes through the point (1,2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

(1) 5 (2) $\frac{62}{5}$ (3) $\frac{31}{5}$ (4) 10

यदि एक वक्र $y = f(x)$ बिन्दु (1,2) से होकर जाता है तथा $x\frac{dy}{dx} + y = bx^4$ को संतुष्ट करता है, तो b के किस मान के लिए $\int_1^2 f(x)dx = \frac{62}{5}$ है ?

(1) 5 (2) $\frac{62}{5}$ (3) $\frac{31}{5}$ (4) 10

Ans. (4)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$, I.F. $= e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + C$

Passes through (1,2), we get

$2 = \frac{b}{5} + C$ .....(i)

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{c}{x}\right)dx = \frac{62}{5}$

$\Rightarrow \frac{b}{25} \times 32 + C\ln 2 - \frac{b}{25} = \frac{62}{5} \Rightarrow C = 0$ & $b = 10$

Topic :- Area Under Curve

Subtopic:- Areabet two Curve (M143)

Level :- Medium

19. The area of the region : $R = \{(x, y): 5x^2 \le y \le 2x^2 + 9\}$ is:

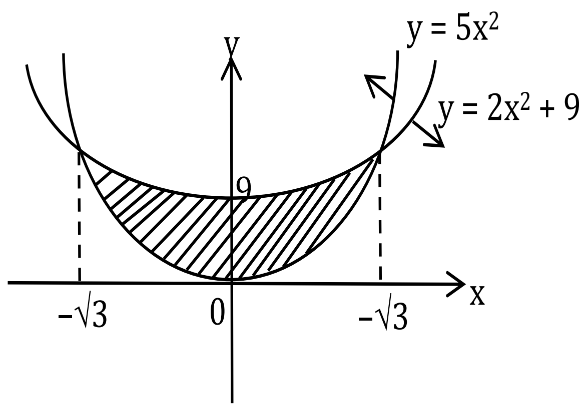
(1) $9\sqrt{3}$ square units (2) $12\sqrt{3}$ square units (3) $11\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

क्षेत्र $R = \{(x, y): 5x^2 \le y \le 2x^2 + 9\}$ का क्षेत्रफल है:

(1) $9\sqrt{3}$ वर्ग इकाई (2) $12\sqrt{3}$ वर्ग इकाई (3) $11\sqrt{3}$ वर्ग इकाई (4) $6\sqrt{3}$ वर्ग इकाई

Ans. (2)

Sol.



Required area

$= 2\int_0^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right)dx$

$= 2\int_0^{\sqrt{3}} \left(9 - 3x^2\right)dx$

$= 2\,|\,9x - x^3\,|_0^{\sqrt{3}} = 12\sqrt{3}$

Topic :- Definite Integration

Subtopic:- Properties (1-4) (M109)

Level :- Medium

20. Let $f(x)$ be a differentiable function defined on [0,2] such that $f'(x) = f'(2-x)$ for all $x \in (0,2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)dx$ is:

(1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1 - e^2)$ (4) $2(1+e^2)$

माना [0,2] में $f(x)$ एक अवकलनीय फलन है जिसके लिए $f'(x) = f'(2-x)$, $\forall x \in (0,2)$, $f(0) = 1$ तथा $f(2) = e^2$ है तो $\int_0^2 f(x)dx$ का मान है:

(1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1 - e^2)$ (4) $2(1+e^2)$

Ans. (1)

Sol. $f'(x) = f'(2-x)$

On integrating both side $f(x) = -f(2-x) + c$

put $x = 0$

$f(0) + f(2) = c \quad \Rightarrow c = 1 + e^2$

$\Rightarrow f(x) + f(2-x) = 1+e^2$ ......(i)

$I = \int_0^2 f(x)\,dx = \int_0^1 \{f(x) + f(2-x)\}\,dx = (1 + e^2)$

Section B

Topic :- Quadratic Equation

Subtopic:- Prob. Based on determinant (M9)

Level :- Medium

1. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is________.

समीकरण $(x+1)^2+|x-5|=\frac{27}{4}$ वास्तविक मूलों की संख्या है________.

Ans. 2

Sol. $x \geq 5$

$$(x+1)^2+(x-5)=\frac{27}{4}$$

$$\Rightarrow x^2+3x-4=\frac{27}{4}$$

$$\Rightarrow x^2+3x-\frac{43}{4}=0$$

$$\Rightarrow 4x^2+12x-43=0$$

$$x=\frac{-12\pm\sqrt{144+688}}{8}$$

$$x=\frac{-12\pm\sqrt{832}}{8}=\frac{-12\pm 28.8}{8}$$

$$=\frac{-3\pm 7.2}{2}$$

$$=\frac{-3+7.2}{2},\frac{-3-7.2}{2}$$ (Therefore no solution)

For $x \leq 5$

$$(x+1)^2-(x-5)=\frac{27}{4}$$

$$x^2+x+6-\frac{27}{4}=0$$

$$4x^2+4x-3=0$$

$$x=\frac{-4\pm\sqrt{16+48}}{8}$$

$x=\frac{-4\pm8}{8}\Rightarrow x=-\frac{12}{8},\frac{4}{8}$

$\therefore$ 2 Real Root's

Topic :- P & C

Subtopic:- Mixed (M224)

Level :- Tough

2. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is________.

छात्रों $S_1, S_2, \ldots, S_{10}$ को तीन समूहों A, B तथा C में इस प्रकार विभाजित करना है कि प्रत्येक समूह में कम से कम एक छात्र हो तथा समूह C में अधिक से अधिक 3 छात्र हों। तो इस प्रकार समूह बनाने की कुल संभावनायें हैं ________.

Ans. 31650

Sol.

$C\rightarrow1$ 9 [A, B]

$C\rightarrow2$ 8 [A, B]

$C\rightarrow3$ 7 [A, B]

$= {}^{10}C_1\,[2^9-2] + {}^{10}C_2[2^8-2] + {}^{10}C_3\,[2^7-2]$

$= 2^7[{}^{10}C_1\times4 + {}^{10}C_2\times2 + {}^{10}C_3] - 20 - 90 - 240$

$= 128\,[40 + 90 + 120\,] - 350$

$= (128\times250) - 350$

$= 10[3165] = 31650$

Topic :- Funciton

Subtopic:- Funetional Equaiton (M200)

Level :- Medium

3. If $a+\alpha=1, b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0,$ then the value of the expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is________.

यदि $a+\alpha=1, b+\beta=2$ तथा $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0,$ है, तो $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ बराबर है________.

Ans. **2**

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ....(i)

$x \to \frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ ....(ii)

(i) + (ii)

$(a+\alpha)\left[f(x)+f\left(\frac{1}{x}\right)\right] = \left(x+\frac{1}{x}\right)(b+\beta)$

$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = \frac{2}{1} = 2$

Topic :- Set & Relation

Subtopic:- Central Tendence & Difterention

Level :- Medium

4. If the variance of 10 natural numbers $1,1,1,...,1,k$ is less than 10, then the maximum possible value of k is________.

4. यदि दस धन पूर्णांकों $1,1,1,...,1,k$ का प्रसरण 10 से कम है, तो k का अधिकतम संभावित मान है ________.

Ans. **11**

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\sigma^2 = \frac{(9+k^2)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$(90 + k^2)\,10 - (81 + k^2 + 8k) < 1000$

$90 + 10k^2 - k^2 - 18k - 81 < 1000$

$9k^2 - 18k + 9 < 1000$

$(k-1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

Topic :- 3D

Subtopic:- Mixed (M178)

Level :- Medium

5. Let λ be an integer. If the shortest distance between the lines $x-\lambda = 2y-1 = -2z$ and $x = y+2\lambda = z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

माना λ एक पूर्णांक है। यदि रेखाओं $x-\lambda = 2y-1 = -2z$ तथा $x = y+2\lambda = z-\lambda$ के बीच की न्यूनतम दूरी $\frac{\sqrt{7}}{2\sqrt{2}}$ है, तो $|\lambda|$ बराबर है

Ans. 1

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$\frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{2}{-1}$ .....(1) Point on line = $\left(\lambda, \frac{1}{2}, 0\right)$

$\frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$ .....(2) Point on line = $(0, -2\lambda, \lambda)$

Distance between skew lines = $\dfrac{\left[\vec{a}_2 - \vec{a}_1\ \vec{b}_1\ \vec{b}_2\right]}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$$\frac{\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}$$

$= \dfrac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}} = \dfrac{\sqrt{7}}{2\sqrt{2}}$ (given)

$= |10\lambda + 3| = 7 \Rightarrow \lambda = -1$

$\Rightarrow |\lambda| = 1$

Topic :- Complex Number

Subtopic:- Eular's form (M281)

Level :- Medium

6. Let $i = \sqrt{-1}$. If $\dfrac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \dfrac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and $n = [|k|]$ be the greatest integral part of |k|. Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to______.

माना $i = \sqrt{-1}$ है। यदि $\dfrac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \dfrac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, है, तथा $n = [|k|]$, |k| का महत्तम पूर्णांक भाग है, तो $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ बराबर है ______.

Ans. 310

Sol. $\dfrac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}} + \dfrac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}$

$\Rightarrow \dfrac{2^{21}.e^{i14\pi}}{2^{12}.e^{-i6\pi}} + \dfrac{2^{21}\left(e^{i7\pi}\right)}{2^{12}\left(e^{i6\pi}\right)}$

$\Rightarrow 2^9 e^{i(20\pi)} + 2^9 e^{i\pi}$

$\Rightarrow 2^9 + 2^9(-1) = 0$

n= 0

$\sum_{j=0}^{5}(j+5)^2 - \sum_{j=0}^{5}(j+5)$

$\Rightarrow \left[5^2+6^2+7^2+8^2+9^2+10^2\right]-\left[5+6+7+8+9+10\right]$

$\Rightarrow \left[\left(1^2+2^2+....+10^2\right)-\left(1^2+2^2+3^2+4^2\right)\right]-\left[\left(1+2+3+.....+10\right)-\left(1+2+3+4\right)\right]$

$\Rightarrow (385-30)-[55-10]$

$\Rightarrow$ 355 – 45 $\Rightarrow$ 310 ans.

Topic :- St. Line

Subtopic:- Locus & (M80)

Level :- Medium

7. Let a point P be such that its distance from the point (5,0) is thrice the distance of P from the point $(-5,0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to

माना एक बिन्दु P इस प्रकार है कि इसकी बिन्दु (5,0) से दूरी, बिन्दु $(-5,0)$ से दूरी का तीन गुना है। यदि बिन्दु P का बिन्दुपथ एक वृत्त है जिसकी त्रिज्या r है, तो $4r^2$ बराबर है______.

Ans. 56

Sol. Let P(h,k)

Given

PA = 3PB

$PA^2 = 9PB^2$

$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2+k^2]$

$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$

$\therefore$ Locus

$x^2 + y^2 + \left(\dfrac{25}{2}\right)x + 25 = 0$

$\therefore c \equiv \left(\dfrac{-25}{4}, 0\right)$

$\therefore\ r^2 = \left(\frac{-25}{4}\right)^2 - 25$

$= \frac{625}{16} - 25$

$= \frac{225}{16}$

$\therefore\ 4r^2 = 4 \times \frac{225}{16} = \frac{225}{4} = 56.25$

After Round of $4r^2 = 56$

Topic :- Binomial Theorem

Subtopic:- Collection of binomial coeff. (M28)

Level :- Tough

8. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \ge r \ge 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to________.

पूर्णांकों n तथा r, माना $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{यदि } n \ge r \ge 0 \\ 0, & \text{अन्य था} \end{cases}$

तो k का वह अधिकतम मान, जिसके लिए, योगफल

$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$ का अस्तित्व है, __________ है।

Ans. Bonus

Sol. $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \ldots\ldots + {}^{10}C_{10}x^{10}$

$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \ldots.{}^{15}C_{k-1}\,x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1} + \ldots\ldots\ {}^{15}C_{15}x^{15}$

$\sum_{i=0}^{k}(10C_i)(15C_{k-i}) = {}^{10}C_0.\ {}^{15}C_k + {}^{10}C_1.{}^{15}C_{k-1} + \ldots.. + {}^{10}C_k.{}^{15}C_0$

Coefficient of x_k in $(1+x)^{25}$

$= {}^{25}C_k$

$\sum_{i=0}^{k+1}\left({}^{12}C_i\right)\left({}^{13}C_{k+1-i}\right) = {}^{12}C_0 . {}^{13}C_{k+1} + {}^{12}C_1 . {}^{13}C_k + \ldots\ldots + {}^{12}C_{k+1} . {}^{13}C_0$

Coefficient of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_{k+1}$

${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

As ${}^nC_r$ is defined for all values of n as will as r. so ${}^{26}C_{k+1}$ always exist

Now k is unbounded so maximum values is not defined.

Topic :- S & P (Progression)

Subtopic:- GP (M17)

Level :- Medium

9. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is__________.

एक गुणोत्तर श्रेणी के पहले चार पदों का योग $\frac{65}{12}$ है तथा उनके व्युत्क्रमों का योग $\frac{65}{18}$ है। यदि इसके पहले तीन पदों का गुणनफल 1 हो और तीसरा पद α, हो तो 2α बराबर है __________.

Ans. 3

Sol. a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$ ....(1)

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$\frac{1}{a}\left(\frac{r^3 + r^2 + r + 1}{r^3}\right) = \frac{65}{18}$ ....(2)

$\frac{(i)}{(ii)}, a^2r^3 = \frac{18}{12} = \frac{3}{2}$

$a^3r^3 = 1 \Rightarrow a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$

$\frac{4}{9}r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$

$\alpha = ar^2 = \frac{2}{3}\cdot\left(\frac{3}{2}\right)^2 = \frac{3}{2}$

$2\alpha = 3$

Topic :- Circle

Subtopic:- Tangent & normal (M97)

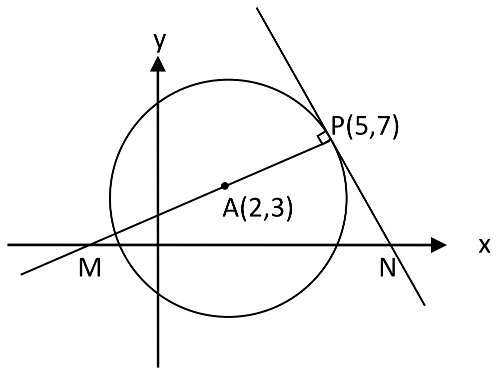
Level :- Medium

10. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5,7) is A, then 24A is equal to_______.

यदि त्रिभुज, जो धनात्मक x-अक्ष तथा वृत्त $(x-2)^2+(y-3)^2=25$ के बिन्दु (5,7) पर खींचे गए अभिलम्ब तथा स्पर्श रेखा द्वारा बनाता है, का क्षेत्रफल A है, तो 24A बराबर है _______.

Ans. Bonus

Sol.



Equation of normal at P

$(y-7) = \left(\frac{7-3}{5-2}\right)(x-5)$

3y −21 = 4x − 20

$\Rightarrow$ 4x − 3y + 1 = 0 ....(i)

$\Rightarrow \quad M\left(-\frac{1}{4}, 0\right)$

Equation of tangent at P

$(y-7) = -\frac{3}{4}(x-5)$

4y −28 = −3x + 15

$\Rightarrow$ 3x + 4y = 43 ....(ii)

$\Rightarrow \quad N\left(\frac{43}{3}, 0\right)$

Hence ar (ΔPMN) = $\frac{1}{2} \times MN \times 7$

$A = \frac{1}{2} \times \frac{175}{12} \times 7$

$\Rightarrow 24A = 1225$

As positive x- axis is given in the question so question should be bonus.

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